Efficient one-step generation of large cluster states with solid-state circuits

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Highly entangled states called cluster states are a universal resource for measurement-based quantum computing (QC). Here we propose an efficient method for producing large cluster states using superconducting quantum circuits. We show that a large cluster state can be efficiently generated in just one step by turning on the inter-qubit coupling for a short time. Because the inter-qubit coupling is only switched on during the time interval for generating the cluster state, our approach is also convenient for preparing the initial state for each qubit and for implementing one-way QC via single-qubit measurements. Moreover, the cluster state is robust against parameter variations.

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Quantum computing (QC) with highly entangled states, known as cluster states, takes advantage of both entanglement and measurement in a remarkable way [1, 2, 3, 4, 5, 6]. In sharp contrast to conventional QC, which uses unitary one- and two-qubit logic operations, this new type of QC is performed through only single-qubit projective measurements on a cluster state. This measurement-based QC is termed "one-way" because it proceeds in an inherently time-irreversible manner. Moreover, it is universal in the sense that any quantum circuit and quantum gates can be implemented on a suitable cluster state [1].

For one-way QC, the initial cluster state should be first generated. This highly entangled state provides a universal resource for QC. Ideally, it is desirable to produce a cluster state in just one step on a scalable circuit, so as to have efficient QC. However, this is challenging. Recently, a quantum-optics experiment [7] implemented one-way QC through local non-deterministic Bell measurements. Even though the cluster state was generated in one step, its generation probability was extremely low. Moreover, it is hard to implement scalable QC with optical cluster states due to the difficulty of large-scale integration in the optical devices. Alternatively, solid-state QC with cluster states were proposed [8, 9] using the Heisenberg exchange interaction between electron spins in quantum dots. In these approaches, additional rotations are performed on individual qubits in order to obtain an effective Ising-like Hamiltonian for producing the cluster state. Also, several steps, instead of the ideal one step, are required to achieve a quantum-dot cluster state.

Here we propose an efficient method for one-step generation of large cluster states using superconducting quantum circuits. These circuits are based on Josephson junctions (JJs) and are regarded as promising candidates of solid-state qubits (see, e.g., [10]). We consider two scalable quantum circuits in which an *inductive coupling* is employed to couple nearest-neighbor charge qubits in one circuit [Fig. 1(a)] and arbitrarily separated charge qubits in the other circuit [Fig. 1(b)]. Both circuits give rise to an Ising-like Hamiltonian, but the inter-qubit interactions are nearest-neighbor and long-range, respectively.

Because the decoherence time for quantum states is limited, in order to have efficient one-way QC, it is essential to generate a cluster state in a deterministic and fast way, so that only a short time is consumed. Also, it should be convenient to prepare the initial state for each qubit and to perform local single-qubit measurements. Furthermore, for a solid-state system, the produced cluster state should be robust against unavoidable parameter variations. Our proposed JJ circuits meet these requirements. First, the cluster state is generated in just one step by turning on, for a short time, the inter-qubit coupling. Also, this cluster-state generation is deterministic, in sharp contrast to the extremely low probability of generating non-deterministic optical cluster states [7]. Second, because the inter-qubit coupling is turned on only when generating the cluster state, the preparation of the initial state for each qubit can be easily achieved, before generating the cluster state, via local single-qubit operations. Moreover, due to the absence of inter-qubit coupling after generating the cluster state, local projective measurements used for one-way QC can also be conveniently performed via single qubits. Third, the cluster state is robust against unavoidable parameter variations because its decoherence time is not sensitive to such parameter variations. Furthermore, all qubits work at the optimal point (namely, the degenercy point) where the quantum state has a longer decoherence time.

Qubit array with nearest-neighbor interactions.— Consider a chain of qubits described by the Hamiltonian

$$H = \hbar g(t) \sum_{i,j} \Gamma(i-j)\, \frac{1 \pm \sigma_x^{(i)}}{2} \frac{1 \pm \sigma_x^{(j)}}{2}, \tag{1}$$

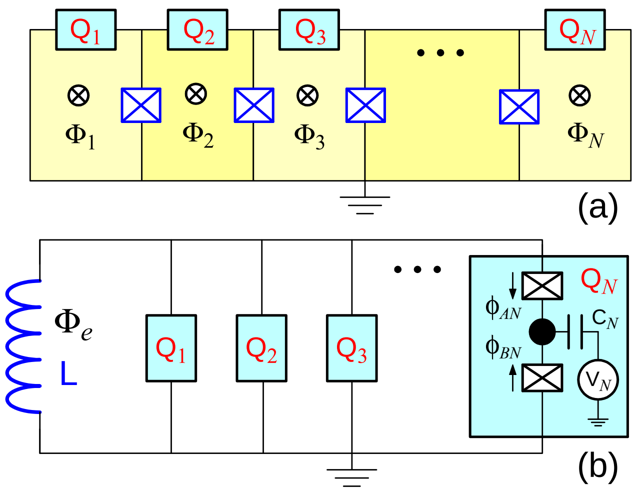

FIG. 1: (Color online) Schematic diagrams of two arrays of superconducting charge qubits (blue boxes). (a) Qubit array $(Q_1, Q_2, \ldots, Q_N)$ with every nearest-neighbor charge qubits coupled by a large Josephson junction (JJ), shown as a crossed rectangle, that acts as an effective inductance L_J. The inter-qubit coupling is induced by the externally applied magnetic flux Φ_i in each small superconducting loop (yellow) connecting qubit Q_i and one or two large JJs. For simplicity, all large JJs are assumed to have the same Josephson coupling energy E_{J0}. (b) Qubit array with long-range interactions, where all charge qubits are connected in parallel to a common inductance L. The inter-qubit coupling is induced by the external magnetic flux Φ_e through the inductance L. As a typical example, we explicitly show the schematic diagram of charge qubit Q_N, where a superconducting island (denoted as a solid circle) is connected to two JJs (with phase drops ϕ_{AN} and ϕ_{BN}) and biased by a voltage V_N through the gate capacitor C_N.

where $\Gamma(i-j)$ specifies the interaction range of the qubits. Similar to the quantum Ising model used for producing cluster states [1, 2], this Hamiltonian is also Ising-like, but its anisotropic direction and the "magnetic" field are along the x direction, instead of the usual z direction. Below we first focus on a chain of superconducting charge qubits with nearest-neighbor interactions.

As shown in [11], two charge qubits can be coupled by a shared inductance. Because a JJ can behave like an effective inductance, one can also replace the common inductance with a large JJ [12, 13, 14]. Figure 1(a) shows an array of charge qubits with a large JJ connected to every pair of nearest-neighbor qubits. This large JJ directly couples the nearest-neighbor charge qubits. Also, the non-nearest-neighbor qubits can be coupled via the large JJs, but the interactions are negligibly small. Here we use the charge states $|0\rangle$ and $|1\rangle$ as the basis states, which correspond to zero and one extra Cooper pairs in the superconducting island of each qubit. The Hamiltonian of the charge-qubit array can be reduced to $H_A = \sum_{i=1}^{N}[H_i + \Lambda_{i,i+1}\sigma_x^{(i)}\sigma_x^{(i+1)}]$, with $\Lambda_{N,N+1} = 0$. The Hamiltonian H_i of the ith charge qubit is $H_i = \varepsilon_i(V_i)\sigma_z^{(i)} - \overline{E}_{Ji}\sigma_x^{(i)}$, with $\varepsilon_i(V_i) = \frac{1}{2}E_{ci}(C_iV_i/e - 1)$ and $\overline{E}_{Ji} = E_{Ji}\cos(\pi\Phi_i/\Phi_0)$. We assume that the charge qubit works in the charging regime with $E_{ci} \gg E_{Ji}$. Here E_{ci} is the charging energy of the superconducting island in the ith qubit and E_{Ji} is the Josephson coupling energy of the two identical JJs coupled to the island; V_i is the gate voltage applied to the qubit and Φ_i is the externally applied magnetic flux through a small loop connecting the ith qubit and one or two large JJs [see Fig. 1(a)]. The flux-dependent inter-qubit coupling is given by [12]

$$\Lambda_{i,i+1} = L_J\left(\frac{\pi^2 E_{Ji}E_{J,i+1}}{\Phi_0^2}\right)\sin\left(\frac{\pi\Phi_i}{\Phi_0}\right)\sin\left(\frac{\pi\Phi_{i+1}}{\Phi_0}\right), \tag{2}$$

where the large JJ acts as an effective inductance $L_J = \Phi_0/2\pi I_0$, with $I_0 = 2\pi E_{J0}/\Phi_0$ being its critical current.

When each charge qubit is shifted to work at the degeneracy point $C_iV_i/e = 1$, the Hamiltonian of the charge-qubit array becomes $H_A = \sum_{i=1}^{N}[-\overline{E}_{Ji}\,\sigma_x^{(i)} + \Lambda_{i,i+1}\,\sigma_x^{(i)}\sigma_x^{(i+1)}]$. Let $\frac{1}{2}\overline{E}_{Ji} = \Lambda_{i,i+1} \equiv \frac{1}{4}\hbar g$ (for $i = 2, 3, \ldots, N-1$), and $\overline{E}_{J1} = \Lambda_{12} = \overline{E}_{JN} \equiv \frac{1}{4}\hbar g$. These conditions can be readily satisfied by choosing suitable E_{Ji} and Φ_i, because $\overline{E}_{Ji}$ decreases from E_{Ji} to zero and $\Lambda_{i,i+1}$ increases from zero to $(\pi/\Phi_0)^2 L_J E_{Ji}E_{J,i+1}$ for $0 < \Phi_i/\Phi_0 < \frac{1}{2}$. The reduced Hamitonian can be written as

$$H_A = \hbar g\sum_{i=1}^{N-1}\frac{1-\sigma_x^{(i)}}{2}\frac{1-\sigma_x^{(i+1)}}{2}, \tag{3}$$

which has the form of Eq. (1) with nearest-neighbor interactions $\Gamma(i-j) = \delta_{i+1,j}$.

Initially, the external flux is not applied, so that no inter-qubit coupling is induced and one can manipulate each charge qubit separately. We first prepare all qubits in the state $|0\rangle_i$. This initial state can be produced by applying a gate voltage to the left ($C_iV_i/e \sim 0$) of the degeneracy point and it corresponds to the ground state of the system. Then, shift the gate voltage V_i fast to the degeneracy point ($C_iV_i/e = 1$) and turn on the externally applied magnetic flux Φ_e to trigger the inter-qubit coupling for a period of time t. The unitary transformation generated by the Hamiltonian (3) is $U(t) = \exp(-iH_A\,t/\hbar)$. The inital state of each charge qubit can be written as $|0\rangle_i = (|-\rangle_i + |+\rangle_i)/\sqrt{2}$, where $|\pm\rangle_i = (|0\rangle_i \mp |1\rangle_i)/\sqrt{2}$ are eigenstates of $H_i = -\overline{E}_{Ji}\sigma_x^{(i)}$ with eigenvalues $\pm\overline{E}_{Ji}$. For the values $gt = (2n+1)\pi$, where n is an integer, the generated state of the charge-qubit array is a highly entangled cluster state:

$$|\phi_N\rangle = \frac{1}{2^{N/2}}\bigotimes_{i=1}^{N}\left(|-\rangle_i + |+\rangle_i\,\sigma_x^{(i+1)}\right), \tag{4}$$

with the convention $\sigma_x^{(N+1)} \equiv 1$.

The generation of cluster states in an array of *capacitively* coupled charge qubits was proposed in [15]. Because of the limitation due to the capacitive inter-qubit interaction, the approach in [15] is valid when each qubit works far away from the degeneracy point. This is not desirable because the decoherence time of a charge qubit becomes much shorter away from the degeneracy point. Furthermore, because the capacitive inter-qubit coupling is fixed [16], it is difficult to prepare the initial state for each qubit. However, the generation of cluster states proposed here employs an array of *inductively* coupled charge qubits. This new proposal has obvious advantages: (1) Each charge qubit works *at* the degeneracy point when generating a cluster state, where the qubit has a *longer* decoherence time; (2) the initial state of all qubits can be easily prepared by turning off the external magnetic flux and shifting the gate voltage away from the degeneracy point; (3) when the initial state is prepared, the cluster state can be readily generated by applying the external flux Φ_i for a period of time; (4) After generating the cluster state, no external magnetic flux is applied and the inter-qubit coupling is switched off. This becomes convenient for implementing one-way QC via local single-qubit measurements on the generated cluster state.

Qubit array with long-range interactions.—When multiple charge qubits are connected to a commonly shared inductance [see Fig. 1(b)], not only nearest-neighbor but also distant qubits can be coupled by this common inductance [11]. Because the common inductance for coupling the charge qubits has a large value ($L \sim 10$ nH) [11], if the circuit is not too large, the inductances of the circuit, except L, can be neglected. The reduced Hamiltonian of the system is given by $H_B = \sum_{i=1}^{N} H_i - \sum_{i,j(j>i)}^{N} \Lambda_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$. Here $\overline{E}_{Ji}$ in the single-qubit Hamiltonian H_i becomes $\overline{E}_{Ji} = E_{Ji}\cos(\pi\Phi_e/\Phi_0)$, with Φ_e being the externally applied magnetic flux through the common inductance L. The inter-qubit coupling is

$$\Lambda_{i,j} = L\left(\frac{\pi^2 E_{Ji}E_{Jj}}{\Phi_0^2}\right)\sin^2\left(\frac{\pi\Phi_e}{\Phi_0}\right). \tag{5}$$

Let $\overline{E}_{Ji}/(N-1) = \Lambda_{ij} \equiv \frac{1}{4}\hbar g$, for $1 \le i, j \le N$ and $j > i$. This condition can be satisfied using N identical charge qubits and a suitable Φ_e. While fulfilling this condition and simultaneously having each charge qubit work *at* the degeneracy point, the Hamiltonian becomes

$$H_B = -\hbar g \sum_{i,j(j>i)}^{N} \frac{1+\sigma_x^{(i)}}{2}\frac{1+\sigma_x^{(j)}}{2}, \tag{6}$$

which corresponds to Eq. (1) with long-range interactions.

The initial state of each charge qubit, $|0\rangle_i = (|-\rangle_i + |+\rangle_i)/\sqrt{2}$, is also prepared by both turning off the external flux Φ_e and applying a gate voltage to the left of the degeneracy point. Furthermore, we shift the gate voltage fast to the degeneracy point and apply the flux Φ_e for a period of time t. The unitary transformation given by the Hamiltonian (6) is $U(t) = \exp(-iH_B\, t/\hbar)$. At $gt = (2n+1)\pi$, the generated cluster state is

$$|\psi_N\rangle = \frac{1}{2^{N/2}}\bigotimes_{i=1}^{N}\left(|-\rangle_i\,(-1)^{N-i}\prod_{j=i+1}^{N}\sigma_x^{(j)} + |+\rangle_i\right), \tag{7}$$

which is also a highly entangled state. In Eq. (4), the operator $\sigma_x^{(i+1)}$ acts on the states $|\pm\rangle$ of the $(i+1)$th qubit. However, for the cluster state $|\psi_N\rangle$, the operator $\sigma_x^{(j)}$ acts on the states $|\pm\rangle$ of the qubits $j = i+1, \ldots, N$, with $i = 1, 2, \ldots, N-1$; this is due to the long-range nature of the inter-qubit coupling in Hamiltonian (6).

Parameter variations and robustness of cluster states.—As in other solid-state systems, parameter variations unavoidably occur when fabricating JJ circuits. When the parameters E_{Ji} vary in the charge-qubit array in Fig. 1(a), because $\overline{E}_{Ji} = E_{Ji}\cos(\pi\Phi_i/\Phi_0)$, the condition $\overline{E}_{J1} = \overline{E}_{JN} = \frac{1}{2}\overline{E}_{Ji} = \frac{1}{4}\hbar g$, with $i = 2, 3, \ldots, N-1$, can be satisfied by adjusting the local magnetic fluxes Φ_i. However, if L_J and E_{Ji} vary, the condition $\Lambda_{i,i+1} = \frac{1}{4}\hbar g$ cannot be satisfied. In order to fulfill this condtion, one can connect a current source in parallel to each large JJ and bias the JJ with a current $I_{bi} < I_0$. Now the inter-qubit coupling becomes

$$\Lambda_{i,i+1} = L_{Ji}\left(\frac{\pi^2 E_{Ji}E_{J,i+1}}{\Phi_0^2}\right)\sin\left(\frac{\pi\Phi_i}{\Phi_0} + \frac{1}{2}\gamma_i\right) \times \sin\left(\frac{\pi\Phi_{i+1}}{\Phi_0} - \frac{1}{2}\gamma_i\right), \tag{8}$$

where the effective inductance for each large JJ is $L_{Ji} = \Phi_0/2\pi I_0\cos\gamma_i$, with $\gamma_i = \sin^{-1}(I_{bi}/I_0)$. Here the condition $\Lambda_{i,i+1} = \frac{1}{4}\hbar g$ can be readily satisfied by changing the bias current I_{bi}. As for the charge-qubit array in Fig. 1(b) and the capacitively coupled charge qubits [15], the conditions for obtaining an Ising-like Hamiltonian cannot be fully satisfied for varying qubit parameters. Therefore, the charge-qubit circuit in Fig. 1(a) should be advantageous for suppressing the effects of unavoidable parameter variations.

Below we further show the robustness of the cluster states against parameter variations. According to the Fermi golden rule, the relaxation rate of the ith charge qubit is $\Gamma_1^{(i)} \equiv 1/T_1^{(i)} = \frac{1}{2}A_iS_i(\Omega)$, where $A_i = \overline{E}_{Ji}^2/(\varepsilon_i^2 + \overline{E}_{Ji}^2)$ and $S_i(\omega)$ is the power spectrum of the charge noise dominant in the charge qubit. For a typical Gaussian noise [17], the dephasing factor is $\eta_i(\tau) = B_i\int d\omega S_i(\omega)\frac{\sin^2(\omega\tau/2)}{2\pi(\omega/2)^2}$, with $B_i = \varepsilon_i^2/(\varepsilon_i^2 + \overline{E}_{Ji}^2)$. The dephasing rate $\Gamma_\varphi^{(i)} \equiv 1/T_\varphi^{(i)}$ is defined by $\eta_i(T_\varphi^{(i)}) = 1$. Following the Bloch-Redfield theory (see, e.g., [18]), the decoherence rate $\Gamma_2^{(i)} \equiv 1/T_2^{(i)}$ is $\Gamma_2^{(i)} = \frac{1}{2}\Gamma_1^{(i)} + \Gamma_\varphi^{(i)}$. Because all inter-qubit couplings are switched off after generating a cluster state, the decoherence time T_2 of the cluster state is given by $1/T_2 = \sum_i 1/T_2^{(i)}$. Here all charge qubits work at the degeneracy point $\varepsilon_i \approx 0$, thus $A_i \approx 1 - (\varepsilon_i/\overline{E}_{Ji})^2$ and $B_i \approx (\varepsilon_i/\overline{E}_{Ji})^2$. Obviously, A_i and B_i are weakly affected by the variations of the parameters E_{Ji}. This indicates that the decoherence time T_2 of the cluster state is not sensitive to the parameter variations. Therefore, the cluster state is robust against the unavoidable parameter variations.

Discussion and conclusion.—Each Josephson coupling energy for a charge qubit is typically $E_J/h \sim 10$ GHz (see, e.g., [16]), which corresponds to a switching time $\tau_1 \sim 0.1$ ns for the single-qubit operation. For the charge-qubit array in Fig. 1(a), the inter-qubit coupling is $\Lambda \sim L_J(\pi E_J/\Phi_0)^2$, where $L_J = \Phi_0/2\pi I_0 = (1/E_{J0})(\Phi_0/2\pi)^2$. Choosing, e.g., $E_{J0} = 5E_J$, one obtains $\Lambda/h \sim 0.5$ GHz. Because $\hbar g/4 = \Lambda$, the shortest time to generate the cluster state is $t_s = \pi/g \sim 0.25$ ns, comparable to the switching time τ_1 of the single-qubit operation. For the array of charge qubits coupled by a common inductance L [see Fig. 1(b)], the inter-qubit coupling is $\Lambda \sim L(\pi E_J/\Phi_0)^2$. Using $L = 10$ nH, one has $\Lambda/h \sim 1.1$ GHz. The corresponding shortest time for generating the cluster state is $t_s \sim 0.11$ ns $\approx \tau_1$. Let T_2 be the decoherence time of a qubit. The decoherence time of N weakly coupled qubits can be estimated as $T_2^{(N)} \sim T_2/N$. For a charge qubit with $T_2 \sim 0.5$ μs at the degeneracy point [19], considering an array with $N = 100$ charge qubits, one obtains $T_2^{(N)} \sim 5$ ns. This decoherence time is longer than the shortest time t_s for generating the cluster state. Here the common inductance is chosen to be large (e.g., $L = 10$ nH), but the inter-qubit couplings are still weak, and those couplings are turned off after generating a cluster state. Thus, the effects of L on the decoherence of the cluster state are small. Moreover, these decoherence effects can be further reduced when L is replaced by a large JJ acting as an effective inductance.

For the usual quantum Ising model, where its anisotropic direction and the "magnetic" field are both along the z direction, the basis states used for representing a cluster state are the eigenstates $|0\rangle_i$ and $|1\rangle_i$ of $\sigma_z^{(i)}$. To implement one-way QC, single-qubit projective measurements are performed [1] on the basis states $|\pm\rangle_i$, namely, the eigenstates of $\sigma_x^{(i)}$. In our proposed charge-qubit arrays the reduced Hamiltonian is also Ising-like, but its anisotropic direction and the "magnetic" field are along the x direction, instead of the z direction [1]. Now the cluster state is represented using basis states $|\pm\rangle_i$, instead of $|0\rangle_i$ and $|1\rangle_i$. Correspondingly, the single-qubit projective measurements are performed on the basis states $|0\rangle_i$ and $|1\rangle_i$. In charge qubits, these states correspond to zero and one extra Cooper pairs in the superconducting island of each qubit. The local single-qubit projective measurements on these basis states can be implemented using, e.g., either a probe junction [16] connected to the superconducting island or a single-electron transistor [20] coupled to the charge qubit. Because the single-electron transistor has high efficiency for reading out the quantum state, it is more advantageous to use it for performing local single-qubit projective measurements. Also, because such a transistor is coupled to each charge qubit via a small capacitance, it only produces weak backaction on the qubit state in the absence of quantum measurement.

In conclusion, we propose an efficient method for producing large cluster states using superconducting circuits. We consider two charge-qubit arrays where either nearest-neighbor or arbitrarily separated qubits are inductively coupled. The initial cluster state can be efficiently generated in just one step by turning on the inter-qubit coupling for a short time. Also, our approach is convenient for preparing the initial state of the system and for implementing one-way QC via single-qubit measurements on the cluster state because the inter-qubit coupling is switched off in both cases.

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